Chapter 24

Alternating Current Circuits
Objective of Lecture

• Generators and Motors
• Inductance
• RL Circuits (resistance and inductance)
• Transformers
• AC

• REMINDER: WORK ON THE EXAMPLES
• Read physics in perspective
Next Chapter

• Chapter 25: Electromagnetic Waves
Electric Motor

• An electromagnet is the basis of an electric motor.
• An electric motor is all about magnets and magnetism: A motor uses magnets to create motion.
• Opposites attract and likes repel. Inside an electric motor, these attracting and repelling forces create rotational motion.
• A motor is consist of two magnets.
Electric Motors:
Motion of a current-carrying loop in a magnetic field

Commutator (rotates with coil)
Vertical position of the loop:
A generator is the opposite of a motor – it transforms mechanical energy into electrical energy. This is an ac generator:

The axle is rotated by an external force such as falling water or steam. The brushes are in constant electrical contact with the slip rings.
Recall in Faraday’s induction experiment, a changing current in one coil induces a current in another coil. This type of interaction between coils is referred to as mutual inductance.

The direction of the induced emf is given by Lenz’s law, which says that the induced current opposes the change that caused it.
Inductance

Inductance is the proportionality constant that tells us how much emf will be induced for a given rate of change in current:

\[ |\mathcal{E}| = N \left| \frac{\Delta \Phi}{\Delta t} \right| = L \left| \frac{\Delta I}{\Delta t} \right| \]

SI unit: \( 1 \text{ V} \cdot \text{s/A} = 1 \text{ henry} = 1 \text{ H} \)

Solving for \( L \),

\[ L = N \left| \frac{\Delta \Phi}{\Delta I} \right| \]
Inductance

Given the definition of inductance, the inductance of a solenoid can be calculated:

\[ L = \mu_0 \left( \frac{N^2}{\ell} \right) A = \mu_0 n^2 A \ell \]

When used in a circuit, such a solenoid (or other coil) is called an inductor.
When the switch is closed, the current immediately starts to increase. The back emf in the inductor is large, as the current is changing rapidly. As time goes on, the current increases more slowly, and the potential difference across the inductor decreases.

R is resistor and L inductor.
This shows the current in an *RL* circuit as a function of time.

The characteristic time is:

\[ \tau = \frac{L}{R} \]

At time \( t = \tau \) the current has risen to 63% of its final value.

- The time constant for an RL circuit depends on both \( L \) and \( R \)
- For a given value of \( R \), \( \tau \) increases with \( L \) as expected.
- Current changes more rapidly with smaller values of \( L \), also as expected.
The LR circuit reminds us of

1. Why physics class is so beautiful
2. Why physics class is so terrible
3. RC circuits
4. Money in the bank
The RC circuit

The voltage across the capacitor is not instantaneously equal to that of the voltage across the battery when the switch is closed. The voltage on the capacitor builds up as more and more charges flows onto the capacitor until the battery is no longer able to "push" any more charge onto the capacitor, at which point the capacitor becomes fully charged.

\[ RC \text{ Circuit, } r = 10000\Omega, \ c = 100\, \mu F \]

R is resistor and C is capacitor
Energy Stored in a Magnetic Field

It takes energy to establish a current in an inductor; this energy is stored in the inductor’s magnetic field.

Considering the emf needed to establish a particular current, and the power involved, we find:

\[ U = \frac{1}{2} LI^2 \]

SI unit: joule, J
Energy Stored in a Magnetic Field

We know the inductance of a solenoid; therefore, the magnetic energy stored in a solenoid is:

\[ U = \frac{1}{2 \mu_0} B^2 A \ell \]

Dividing by the volume to find the energy density gives:

\[ u_B = \frac{\text{magnetic energy}}{\text{volume}} = \frac{B^2}{2\mu_0} \]

This result is valid for any magnetic field, regardless of source.
WORK HOME: stored energy

A 24V battery is connected in series with a resistor and an inductor, where \( R = 8.0 \Omega \) and \( L = 4.0 \text{H} \). Find the energy stored in the inductor (a) when the current reaches its maximum value and (b) one time constant after the switch is closed.
Getting Power to Our Homes

• Let’s power our homes with DC power
  – DC means direct current: just like what batteries deliver
• But want power plants far from home
  – and ability to “ship” electricity across states
• So power lines are long
  – resistance no longer negligible

looks like:
DANGER!

• But having high voltage in each household is a recipe for disaster
  – sparks every time you plug something in
  – risk of fire
  – not cat-friendly

• Need a way to step-up/step-down voltage at will
  – can’t do this with DC, so go to AC
A way to provide high efficiency, safe low voltage:

High Voltage Transmission Lines

Low Voltage to Consumers

step-up to 500,000 V

step-down, back to 5,000 V

≈5,000 Volts

step-down to 120 V
Transmission structures

Typical Electric Line Structures

- 500 kv
- 230 kv
- 138 kv
- 69 kv
- 7 - 13 kv

500,000        230,000        138,000       69,000       7–13,000
long-distance     neighborhood

three-phase “live” wires
to house
A transformer is used to change voltage in an alternating current from one value to another.
Transformers and Transmission of Power

This is a step-up transformer – the emf in the secondary coil is larger than the emf in the primary:
Transformers

By applying Faraday’s law of induction to both coils, we find:

\[
\frac{V_p}{V_s} = \frac{N_p}{N_s}
\]

Here, \( p \) stands for the primary coil and \( s \) the secondary.
Transformers

The power in both circuits must be the same; therefore, if the voltage is lower, the current must be higher.

**Transformer Equation (Current and Voltage)**

\[
\frac{I_s}{I_p} = \frac{V_p}{V_s} = \frac{N_p}{N_s}
\]
Example 1

• A disk drive plugged into a 120-V outlet operates on a voltage of 9.0 V. The transformer that powers the disk drive has 125 turns on its primary coil. (a) Should the number of turns on the secondary coil be the same, greater that or less than 125? Explain  (b) Find the number of turn on the secondary coil.
A disk drive plugged into a 120-V outlet operates on a voltage of 9.0 V. The transformer that powers the disk drive has 125 turns on its primary coil. (a) Should the number of turns on the secondary coil be the same, greater that or less than 125?

1. same 0%
2. Greater than 0%
3. Less than 0%
4. I am confused 0%
Alternating Voltages and Currents

Wall sockets provide current and voltage that vary sinusoidally with time.

Here is a simple ac circuit:
Alternating Current (AC) vs. Direct Current (DC)

• AC is like a battery where the terminals exchange sign periodically!
• AC sloshes back and forth in the wires
• Although net electron flow over one cycle is zero, can still do useful work!
  – Imagine sawing (back & forth), or rubbing hands together to generate heat
Alternating Voltages and Currents

The voltage as a function of time is:

\[ V = V_{\text{max}} \sin \omega t \]
Alternating Voltages and Currents

Since this circuit has only a resistor, the current is given by:

\[ I = \frac{V}{R} = \left( \frac{V_{\text{max}}}{R} \right) \sin \omega t = I_{\text{max}} \sin \omega t \]

Here, the current and voltage have peaks at the same time – they are in phase.
In order to visualize the phase relationships between the current and voltage in ac circuits, we define phasors – vectors whose length is the maximum voltage or current, and which rotate around an origin with the angular speed of the oscillating current.

The instantaneous value of the voltage or current represented by the phasor is its projection on the y axis.
Resistor in an AC Circuit

- Consider a circuit consisting of an AC source and a resistor
- The graph shows the current through and the voltage across the resistor
- The current and the voltage reach their maximum values at the same time
- The current and the voltage are said to be in phase

Voltage varies as

$$\Delta V = \Delta V_{\text{max}} \sin 2\pi ft$$

Same thing about the current

$$I = I_{\text{max}} \sin 2\pi ft$$
More About Resistors in an AC Circuit

• The direction of the current has no effect on the behavior of the resistor

• The rate at which electrical energy is dissipated in the circuit is given by
  
  \[ P = i^2 R \]

  • where \( i \) is the *instantaneous current*
  
  • the heating effect produced by an AC current with a maximum value of \( I_{\text{max}} \) is not the same as that of a DC current of the same value
  
  • The maximum current occurs for a small amount of time
24-1 Alternating Voltages and Currents

By calculating the power and finding the average, we see that:

\[ P = I^2R = I_{\text{max}}^2R \sin^2 \omega t \]

\[ P_{\text{av}} = I_{\text{max}}^2R(\sin^2 \omega t)_{\text{av}} = \frac{1}{2} I_{\text{max}}^2R \]

\[ = I_{\text{rms}}^2R \]
Electrical Power Transmission

• When transmitting electric power over long distances, it is most economical to use high voltage and low current
  – Minimizes $I^2R$ power losses
• In practice, voltage is stepped up to about 230,000 V at the generating station and stepped down to 20,000 V at the distribution station and finally to 120 V at the customer’s utility pole
Estimate resistance of power lines:
say 0.001 Ohms per meter, times 200 km = 0.001 \Omega/m \times 2 \times 10^5 m = 20 Ohms

We can figure out the current required by a single bulb using
\[ P = VI \] so
\[ I = \frac{P}{V} = \frac{120 \text{ Watts}}{12 \text{ Volts}} = 10 \text{ Amps} \]

Power in transmission line is
\[ P = I^2R = 10^2 \times 20 = 2,000 \text{ Watts} \]

“Efficiency” is
\[ \epsilon = \frac{P}{P_{\text{total}}} = \frac{120 \text{ Watts}}{4120 \text{ Watts}} = 0.3\% \]

What could we change in order to do better?
The Tradeoff

• The thing that kills us most is the high current through the (fixed resistance) transmission lines

• **Need less current**
  – it’s that square in $I^2R$ that has the most dramatic effect

• But our appliance needs a certain amount of power
  – $P = VI$ so less current demands higher voltage

• Solution is **high voltage** transmission
  – Repeating the above calculation with 12,000 Volts delivered to the house draws only
    
    \[ I = \frac{120 \text{ Watts}}{12 \text{ kV}} = 0.01 \text{ Amps} \text{ for one bulb, giving} \]
    
    \[ P = I^2R = (0.01)^2 \times 20 = 20 \times 10^{-4} \text{ Watts, so} \]
    
    \[ P = 0.002 \text{ Watts of power dissipated in transmission line} \]

**Efficiency in this case is** $\varepsilon = \frac{120 \text{ Watts}}{120.004} = 99.996\%$
Alternating Voltages and Currents

A ground fault circuit interrupter can cut off the current in a short circuit within a millisecond.